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Some Physical Constraints on Gauge Models of Weak Interactions

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ABSTRACT

Recently proposed models of weak interactions based on spontaneously broken gauge symmetry contain additional interactions arising from exchange of the scalar Higgs' particles and/or neutral vector bosons. Further, higher order corrections are finite, and therefore should be taken seriously. We investigate what constraints on parameters of models are imposed by consideration of Higgs' particle exchange and of higher order effects in K decays. To bring out the main points we shall focus mainly on the SO(3) models of Georgi and Glashow.

In the 5-quark version of the Georgi-Glashow model, the Higgs' scalar couples strongly to $(\bar{\lambda}n)$, and to $(\bar{e}e)$, so that processes such as $K^+ \to \pi^+ + e + \bar{e}$ decay can occur already in lowest order. Thus a stringent lower bound ($m_\phi \geq 10$ GeV) is imposed on the mass of the scalar particle. For the process $K_L \to \mu + \bar{\mu}$, which occurs in second order, we find the amplitude to be of order $G_F\alpha \sin\theta_c$ and <u>independent</u> of the value of M_W. This is clearly in contradiction with experiment and rules out the 5-quark version.

We analyze also an 8-quark version of the model, in which extra quarks are used to suppress the amplitudes for $K_L \to \mu + \bar{\mu}$ and $K^o \leftrightarrow \overline{K}^o$: the amplitudes are of order $G_F\alpha\,(\Delta m^2/M_W^2)$ where Δm^2 is the difference between the squared masses of "charmed" and "uncharmed" quarks. It is also shown that in this version single scalar particle

exchange is altogether forbidden and the constraint on m_{ϕ} is accordingly eliminated.

Constraints similar to those found for the Georgi-Glashow models also apply to other spontaneously broken gauge models of weak interactions.

I. INTRODUCTION

The standard phenomenology of weak interactions is usually pictured as arising, effectively, from the self-interaction of a charged V-A current composed of leptonic and hadronic parts. It has recently become evident[1] that this description can be encompassed within the framework of renormalizable field theories, based on the strategy of spontaneously broken gauge symmetry,[2] which unify the weak and electromagnetic interactions. The theoretical possibilities are enormously varied, provided one is allowed to freely invent intermediate vector bosons, Higgs' scalar particles, new heavy leptons, etc. - all sufficiently massive to have so far escaped detection. The experimentally well-established features of weak processes then arise in lowest order, through current-current couplings mediated by charged vector bosons. However, additional interactions, not contemplated in the usual phenomenology, are inevitably introduced in the various models. These arise from exchange of the scalar Higgs' particles, which are an essential feature of the broken gauge symmetry scheme. Moreover, weak neutral currents coupled to neutral vector bosons play a role in a certain subclass of models. Finally, because higher order corrections are finite and are therefore to be taken seriously, it is essential to check whether these corrections are indeed small enough to protect the phenomenology arranged for in lowest order.

On these matters the question of neutral current effects, or of higher order effects which simulate them, are of special interest. For purely leptonic processes, e.g., $\bar{\nu}_e + e \rightarrow \bar{\nu}_e + e$, $\nu_\mu + e \rightarrow \nu_\mu + e$, the upper bounds presently available are only moderately restrictive.[3] The situation for neutral current effects is somewhat more restrictive in the case of strangeness-conserving semi-leptonic processes, e.g., ν + nucleon $\rightarrow$ ν + hadrons; in fact the upper bounds have recently diminished sufficiently to make serious trouble for certain models which feature neutral currents.[4] Most decisive is the situation for neutral current effects in $\Delta S \neq 0$ semi-leptonic processes such as $K_L \rightarrow \bar{\mu}\mu$, $K^+ \rightarrow \pi^+ + e^+ + e^-$, $K^+ \rightarrow \pi^+ + \nu + \bar{\nu}$, and the $K_1 - K_2$ mass difference. The upper bounds are so restrictive that one takes it as a principle of model building to banish strangeness-changing neutral vector boson couplings altogether. Even when this is accomplished, however, it is necessary to see what further model constraints are imposed by consideration of Higgs' particle exchange or higher order effects. These are the issues to be discussed in the present paper.

There is no known theoretical principle in the broken gauge symmetry scheme which relates the masses of Higgs' particles to the masses of other particles in the models. Thus it can always be arranged that observable effects arising from exchange of the Higgs' particles are as small as one wishes. Nevertheless, it may be useful to know what lower limits on mass can be tolerated. On the other hand, higher order effects, arising say from exchange of two charged vector bosons, are less easily suppressed by adjustment of free parameters, at least in certain types of models; so consideration of these effects can be used more decisively to narrow the choice of allowable models. In fact this has already been widely noted, in a qualitative way, and has served to guide the building of models. Our purpose is to suggest what may be a more quantitative basis for making the appropriate estimates.

We do not attempt here to systematically review all of the various models which have been proposed. In order to bring out the main points we shall instead focus mainly on the SO(3) models of Georgi and Glashow,[5] beginning with the "5-quark model". This is the simplest model which avoids weak neutral vector bosons altogether; but, as it turns out, it runs afoul almost unavoidably of the unobserved processes $K_L \to \bar{\mu}\mu$, etc. In this latter connection we improve somewhat on an estimate made by Georgi and Glashow. They noted the danger but suggested that it might be overcome by choosing a small enough mass M_W for the charged vector boson. Already in previous analyses of nonrenormalizable models of the weak interactions,[6] one argued that the amplitude for processes such as $K_L \to \bar{\mu}\mu$ must be of the size $G(G\Lambda^2)$, where Λ is a cutoff mass - which has, then, to be fairly small. In the renormalizable models Λ is typically replaced by a vector boson mass: M_W in the model of Georgi and Glashow. In the latter model, however, the factors are such that $G\Lambda^2$ is replaced by the fine structure constant α, <u>independent</u> of M_W. The resulting amplitude is far too big to be tolerated. We shall also analyze an "8 quark" variant of the model, suggested originally by Bjorken. Here a cancellation effect can be arranged in order to (perhaps) eliminate the trouble. In this respect the "8 quark" model is representative of various other schemes in which $K_L \to \bar{\mu}\mu$ decay is similarly suppressed by cancellation effects; in particular, it is representative of a model of Glashow, Iliopoulos, and Maiani,[7] who first proposed this cancellation mechanism. In schemes of this sort, typically, the amplitude is of order $G\alpha(\Delta m^2/M_W^{\ 2})$, where Δm^2 is the difference between the squared masses of "uncharmed" and "charmed" quarks; and it is possible to imagine that the suppression factor $\Delta m^2/M_W^{\ 2}$ is small enough to be in accord with experimental upper bounds.

Concerning effects arising from exchange of scalar Higgs' particles, the situation is again well illustrated in the two variants of the Georgi-Glashow model. In the 5-quark version processes such as $K^+ \to \pi^+ + e + \bar{e}$ decay can occur already in lowest order and thus a stringent bound ($m_\phi \geq 10$ GeV) is imposed on the mass of the scalar particle. On the other hand, single scalar particle exchange is altogether forbidden for these processes in the 8-quark variant; and the mass constraint is correspondingly eliminated.

II. THE GEORGI-GLASHOW 5-QUARK MODEL

A. Effects of the Higgs' Scalar

In the electron sector the Georgi-Glashow model is based on the particles e, ν_e and two new heavy leptons X^+, X^o; similarly for the muon sector: μ, ν_μ, Y^+, Y^o. On the hadronic side the model is based on integrally charged quarks, p, n, λ and charmed quarks q^o, q^-. We shall assume familiarity with the model and begin by considering Higgs' scalar particle effects.

The coupling of the Higgs' field ϕ with quark fields is determined by quark masses[8] and has the form

$$\mathcal{L}_\phi = e\phi \left\{ \sin^2\beta \sin\theta_c \cos\theta_c \; \bar{\lambda}\left(\frac{m_\lambda + m_n}{2M_W} - \frac{m_\lambda - m_n}{2M_W}\gamma_5\right) n \right. \tag{1}$$

$$\left. + h.c. + \bar{e}e\left(\frac{m_{X^+} - m_e}{2M_W}\right) + \bar{\mu}\mu\left(\frac{m_{Y^+} - m_\mu}{2M_W}\right) + \cdots \right\},$$

where we have displayed only those terms which are relevant in the lowest order of the processes $K^+ \to \pi^+ + e + \bar{e}$, $K_L \to \bar{\mu}\mu$, etc. Here θ_c is the Cabibbo angle and β is a free parameter of the model, related to the vector boson mass M_W according to

$$\frac{e^2 \sin^2\beta}{4M_W^2} = \frac{G}{\sqrt{2}}, \quad \text{or} \quad M_W = (52.8\ \mathrm{GeV}/c^2)\sin\beta. \tag{2}$$

It is clear that the pseudoscalar quark term will be involved in the process $K_L^0 \to \bar{\mu}\mu$ and that the scalar term will contribute to processes like $K^+ \to \pi^+ + \ell + \bar{\ell}$. If $m_\lambda - m_n \ll m_\lambda + m_n$, as we shall suppose, consideration of the latter processes will provide the better constraint on the Higgs' scalar mass m_ϕ.

The strongest experimental limit for $K_{\ell 3}$ "neutral current" processes is

$$\frac{\Gamma(K^+ \to \pi^+ + e^+ + e^-)}{\Gamma(K^+ \to \mathrm{all})} \leq 4 \times 10^{-7}. \tag{3}$$

Let us estimate the contribution of the interaction of Eq. (1). We require the matrix element $\langle \pi^+ | (S)_3^2 | K^+ \rangle$ of the scalar operator $(S)_3^2 = \bar{\lambda} n$. Now for ordinary $K_{\ell 3}$ decay, $K^+ \to \pi^0 + e^+ + \nu$, we encounter the matrix element $\langle \pi^0 | (V_\mu)_3^1 | K^+ \rangle$ of the vector current $(V_\mu)_3^1 = \bar{\lambda}\gamma_\mu p$, and

$$\langle \pi^0 | (V_\mu)_3^1 | K^+ \rangle = \frac{1}{\sqrt{2}} \left[f_+ (p_K + p_\pi)_\mu + f_- (p_K - p_\pi)_\mu \right]. \tag{4}$$

If we suppose that conservation of the vector currents is broken only by mass terms in the strong interaction Lagrangian, then

$$(S)_3^2 = \frac{1}{m_\lambda - m_n}\, i\, \partial^\mu (V_\mu)_3^2 .$$

The matrix elements $<\pi^+ | \partial^\mu (V_\mu)_3^1 | K^+>$ and $<\pi^+ | \partial^\mu (V_\mu)_3^2 | K^+>$ can be simply be related through isospin considerations, and we find

$$\langle \pi^+ | (S)_3^2 | K^+ \rangle = \frac{i}{m_\lambda - m_n} \left[f_+ (m_K^2 - m_\pi^2) + f_- (p_K - p_\pi)^2 \right]. \tag{5}$$

The resulting amplitude for $K^+ \to \pi^+ + e^+ + e^-$ decay is then given by

$$A(K^+ \to \pi^+ + e^+ + e^-) = \frac{G}{\sqrt{2}} \sin\theta_c \cos\theta_c \left(\frac{m_\lambda + m_n}{m_\lambda - m_n}\right) \frac{m(X^+)}{m_\phi} \tag{6}$$

$$\times \left[f_+ m_K^2 + f_- (p_K - p_\pi)^2 \right] \bar{v}_e u_e .$$

For comparison,

$$A(K^+ \to \pi^0 + e^+ + \nu) = \frac{G}{2} \sin\theta_c \, \bar{u}_\nu \left[f_+ \gamma \cdot (p_K + p_\pi) + m_e f_- \right] (1 - \gamma_5) v_e .$$

Making reasonable approximations we then find

$$R_1 = \frac{\Gamma(K^+ \to \pi^+ + e + \bar{e})}{\Gamma(K^+ \to \pi^0 + \bar{e} + \nu)} \simeq 2 \left(\frac{m_\lambda + m_n}{m_\lambda - m_n}\right)^2 \left(\frac{m_K \, m(X^+)}{m_\phi^2}\right)^2 . \tag{7}$$

Experiment requires that $R_1 < 10^{-5}$, or

$$m_\phi \gtrsim \left(\frac{m_\lambda + m_n}{m_\lambda - m_n}\right)^{1/2} (10 \text{ GeV}) \tag{8}$$

where we have assumed $m_{X^+} \gtrsim m_K$, consistent with nonobservation of X^+ in K meson decay or in neutrino-induced reactions. Equation (8) is a much more stringent lower bound on m_ϕ than can be obtained by considering strangeness-conserving interactions - for example, the level shift due to ϕ exchange in muonic atoms.[9]

We recall from previous work[9,10] that the weak correction to muon g-2 is given by

$$\left(\frac{g_\mu - 2}{2}\right)^{weak} = -\frac{\alpha}{8\pi}\frac{m_\mu m(Y^+)}{M_W^2} F\left[\left(m(Y^0)/M_W\right)^2\right]$$

$$+\frac{\alpha}{8\pi}\frac{m_\mu m(Y^+)}{M_W^2}\frac{m_\mu m(Y^+)}{m_\phi{}^2} G\left[\left(m_\mu/m_\phi\right)^2\right],$$

where F(x) and G(x) are positive functions of order unity for reasonable values of their arguments:

$$F(x) = \frac{3}{(1-x)^2}\left(1 - 3x - \frac{2x^2}{1-x}\ln x\right) + 1 ,$$

$$G(x) = \int_0^1 d\lambda \frac{2\lambda^2 - \lambda^3}{\lambda^2 x - \lambda + 1} .$$

From a treatment of $K^+ \to \pi^+ + \mu^+ + \mu^-$ identical to that given above for $K^+ \to \pi^+ + e^+ + e^-$ it follows that

$$\frac{m_\mu m(Y^+)}{m_\phi^2} < 10^{-3}\left(\frac{m_\lambda - m_n}{m_\lambda + m_n}\right),$$

and therefore the second term in the expression for muon g-2 above is negligible. There is no longer the possibility of the first and second terms cancelling. In order for $(g_\mu - 2)$ to remain within the experimental bounds (with two standard deviations):

$$-3 \times 10^{-7} \leq (g_\mu - 2)/2 \leq 9 \times 10^{-7},$$

the intermediate vector boson mass must be rather large in the 5-quark model:

$$M_W \gtrsim 20 \text{ GeV}.$$

B. W^+W^- Intermediate States

Let us next turn to the question of higher order contributions to the (non-observed) processes under discussion here. To simplify the analysis we shall assume that the mass M_W of the charged vector bosons is large compared to the masses of all the quarks and leptons, and we focus on contributions arising from exchange of two vector bosons. We first compute the amplitude for the scattering process $\bar{\lambda}+n \rightarrow \mu^+ + \mu^-$, in the limit of small external momenta. The resulting expression is then employed as an effective Lagrangian in order to provide an estimate for the amplitude describing $K_L \to \bar{\mu}\mu$. In this we are ignoring the possibility of an accidental cancellation between contributions from single Higgs' particle exchange (already discussed) and exchange of two vector bosons. Also, insofar as K mesons (and other familiar bosons) are regarded as bound states formed of quarks, we ignore the possibility of a direct $K_L \to \bar{\mu}\mu$ coupling.

The calculation is simplest in the 't Hooft gauge[1] (equivalently, the R_ξ gauge,[10] with $\xi = 1$). Here the vector boson propagator is $\delta_{\mu\nu}/(k^2 - M_W^2)$. In this gauge, strictly speaking, it is necessary to consider also the exchange of the <u>unphysical</u>, <u>charged</u> Higgs' scalars; but the latter contributions can in fact be ignored insofar as M_W is large compared to all quark and lepton masses. Thus we are left with the vector boson exchange diagrams of Fig. 1. The calculation is now essentially the same one as in Ref. (11). The effective interaction , evaluated for small external momenta, is

$$\frac{3G\alpha}{\sqrt{2}\pi} \sin\theta_c \cos\theta_c \left(\bar{\lambda}\gamma_\alpha\left(\frac{1-\gamma_5}{2}\right)n\right)\left(\bar{\mu}\gamma^\alpha\gamma_5\mu\right) + h.c. \,. \tag{9}$$

For analysis of $K_L \to \bar{\mu}\mu$ decay we require the matrix element $\langle 0|\bar{\lambda}\gamma^\alpha\gamma_5 n|K_L\rangle$. On the basis of simple isospin considerations this can be related to the amplitude $\langle 0|\bar{\lambda}\gamma^\alpha\gamma_5 p|K^+\rangle = f_K p_K^{\ \alpha}$ which describes ordinary $K^+ \to \ell^+ + \nu$ decay. In this way we find

$$A(K_L \to \bar{\mu}\mu) = \frac{3G\alpha}{\sqrt{2}\pi} \sin\theta_c \cos\theta_c \frac{f_K}{\sqrt{2}} 2m_\mu \bar{\mu}\gamma_5\mu. \tag{10}$$

This is to be compared with

$$A(K^+ \to \mu^+ + \nu) = \frac{G}{\sqrt{2}} f_K \sin\theta_c \, m_\mu \, \bar{\nu}\gamma_5 \mu . \tag{11}$$

Thus, to a good approximation

$$\frac{\Gamma(K_L \to \bar{\mu}\mu)}{\Gamma(K^+ \to \mu^+ + \nu)} \simeq \left(\frac{3\sqrt{2}\,\alpha}{\pi}\right)^2 \simeq 10^{-4} , \tag{12}$$

corresponding to

$$R_2 = \frac{\Gamma(K_L \to \bar{\mu}\mu)}{\Gamma(K_L \to all)} \simeq 3 \times 10^{-4} . \tag{13}$$

This result, independent of M_W or other adjustable parameters, is clearly inconsistent with experiment and rules out the 5-quark version of the SO(3) model of Georgi and Glashow.

III. THE GEORGI-GLASHOW 8-QUARK MODEL

A. Constraints from $K_L \to \bar{\mu}\mu$

In order to avoid difficulties with $K_L \to \bar{\mu}\mu$, etc., one has to arrange for cancellations, as first suggested by Glashow, Iliopoulos, and Maiani.[7] In the SU(4) model discussed by them, or in an analogous SU(2) $\times$ U(1) gauge model due to Weinberg,[12] this cancellation is effected by the introduction of a fourth "charmed" quark p', in addition to p, n, λ. The charged vector boson couples the quark p to the Cabibbo neutron quark $n_c = n \cos \theta_c + \lambda \sin \theta_c$, and the quark p' to $\lambda_c = -n \sin \theta_c + \cos \theta_c$, with identical coupling. For the process $\bar{\lambda} + n \rightarrow W^+ + W^-$ one then sees that the contributions from intermediate p and p' states exactly cancel insofar as these particles have the same mass. In the context of the SO(3) gauge scheme of Georgi and Glashow, similar cancellation effects are accomplished by introduction of eight quarks, as described in the Appendix. At the same time, the couplings $\phi \bar{\lambda} n$ and $\phi \bar{\lambda} \gamma_5 n$ of the Higgs scalar vanish altogether in the 8-quark model, as we explain in the Appendix. Consequently, consideration of the processes $K^+ \rightarrow \pi^+ + e^+ + e^-$, $K_L \rightarrow \bar{\mu}\mu$, etc., does not impose any stringent limits on the mass m_ϕ.

The lowest order diagrams describing $\bar{\lambda} + n \rightarrow W^+ + W^-$ in the 8-quark model are shown in Fig. 2. The amplitude is nonvanishing only insofar as the mass differences $\Delta m_p^2 = m_p^2 - m_{p'}^2$ and $\Delta m_q^2 = m_{q^-}^2 - m_{q'^-}^2$ do not vanish. Allowing for such mass differences we proceed to compute the effective Lagrangian for $\bar{\lambda} + n \rightarrow \mu^+ + \mu^-$ as before. Working to leading order in quark mass differences, and for simplicity neglecting $m(Y^o)$ compared to quark masses, we find

$$\frac{G\alpha}{4\sqrt{2}\,\pi} \frac{1}{M_W^2} \ln \frac{M_W^2}{m^2} \sin\theta_c \cos\theta_c \left(\bar{\lambda}\gamma^\alpha(1-\gamma_5)n\right) \times \bar{\mu}\gamma_\alpha \left[5(\Delta m_p^2 - \Delta m_q^2) + 3(\Delta m_p^2 + \Delta m_q^2)\gamma_5\right]\mu, \tag{14}$$

where m^2 is a typical quark mass. Thus the branching ratio for $K_L \to \bar{\mu}\mu$ predicted in the 8-quark model is

$$R_2 = \frac{\Gamma(K_L \to \bar{\mu}\mu)}{\Gamma(K_L \to all)} \simeq (7 \times 10^{-5}) \left(\frac{\Delta m_p^2 + \Delta m_q^2}{M_W^2} \ln \frac{M_W^2}{m^2} \right)^2 . \tag{15}$$

The current experimental upper limit on the branching ratio is $R_2 < 1.8 \times 10^{-9}$.[13] The smallness of this value is of course in itself very interesting, since the 2γ intermediate state alone is expected, on the basis of the measured rate of $K_L \to 2\gamma$, to give a branching ratio of at least $(6 \pm 1) \times 10^{-9}$.[18] Just to be conservative we assume that the actual branching ratio is closer to this "unitarity bound". For simplicity, let us assume also that

$$|\Delta m_p^2| \approx |\Delta m_q^2| \approx 2m(m_{p'} - m_p),$$

with

$$\Delta m \equiv |m_p - m_{p'}| \gtrsim 1 \text{ GeV},$$

since charmed hadrons would presumably have been seen already if they were not roughly one GeV/c^2 heavier than ordinary hadrons.[7,15] It then follows that

$$\frac{2m\Delta m}{M_W^2} < \frac{\Delta m_p^2}{M_W^2} \ln \frac{M_W^2}{m^2} < 5 \times 10^{-3}, \tag{16}$$

or, using Eq. (2),

$$m < (7\ \mathrm{GeV}/c^2)\sin^2\beta \quad . \tag{17}$$

B. $\underline{K_1 - K_2}$ Mass Difference

In order to treat the $K_1 - K_2$ mass difference by the method used for $K_L \to \bar{\mu}\mu$, we must be able to estimate the contribution of an effective interaction which is the product of four hadronic currents. We unfortunately cannot do this with any precision because of the presence of strong interactions. Nevertheless, we do not expect phenomenological estimates to be wrong in order of magnitude.

Calculating the effective interaction as before, we evaluate the graphs of Fig. 3 and obtain

$$\mathcal{L}_{\Delta S=2} \simeq -\frac{G^2}{24\pi^2}\sin^2\theta_c\cos^2\theta_c\left[3(\Delta m_p + \Delta m_q)^2 - 5(\Delta m_p - \Delta m_q)^2\right] \tag{18}$$

$$\times\left[\bar{\lambda}\gamma_\alpha(1-\gamma_5)n\right]\left[\bar{\lambda}\gamma^\alpha(1-\gamma_5)n\right],$$

in the 8-quark model. We must now estimate the matrix element of $\mathcal{L}_{\Delta S=2}$ between $\overline{K}^0$ and K^0. In order to do this, we will make the simplest approximation[6] and just insert the vacuum intermediate state between the two $\Delta S = 1$ currents:

$$\langle \bar{K}^0 | (\bar{\lambda}\gamma^\alpha(1-\gamma_5)n)^2 | K^0 \rangle \approx 4 \langle \bar{K}^0 | (J_\mu)_3^2 | 0 \rangle \langle 0 | (J^\mu)_3^2 | K^0 \rangle$$
$$\simeq 4\,(f_K m_K)^2 / 2m_K\,, \tag{19}$$

where the factor 4 comes from inserting the vacuum state in all possible Fierz orderings of the four fermion operators. Alternative treatments give approximately the same answer.[16] Again assuming Eq. (16), for simplicity, and comparing with the measured value of $\Delta m_K = |m_{K_1} - m_{K_2}|$ we find

$$\frac{\Delta m_K}{m_K} = (7.14 \pm 0.05) \times 10^{-15} \approx 3.2 \times 10^{-14} \left(\frac{\Delta m}{1\,\mathrm{GeV}/c^2}\right)^2, \tag{20}$$

implying a mass difference between charmed and normal quarks of $\Delta m \approx 1/2\ \mathrm{GeV}/c^2$. If we abandon the simplifying assumption $\Delta m_p^2 = \Delta m_q^2$, it is possible to adjust Δm_q^2 to vary the calculated value of Δm_K over a wide range. This fact, coupled with the theoretical uncertainties introduced by the presence of strong interactions, prevent us from drawing any sharp quantitative conclusions from our consideration of $m_{K_1} - m_{K_2}$. We should stress, however, that if we had tried to calculate this quantity in a model which, like the Georgi-Glashow 5-quark model, does not incorporate a cancellation mechanism to provide strong suppression of $\Delta S = 2$ neutral processes, the result would have been unacceptably large.

IV. DISCUSSION

We have analyzed the SO(3) models of Georgi and Glashow as examples of the large range of possible gauge models of weak and electromagnetic interactions. To what extent does our analysis apply to other models?

In order to answer this questions, we have also looked at the $SU(2) \times U(1)$ models, of which the prototype is Weinberg's original gauge model.[1] If hadrons are incorporated in this model in the simplest way, with the doublet (p, n_c) introduced in analogy with $(e, \nu)_L$, then $\Gamma(K_L \to \bar{\mu}\mu)$, $m_{K_1} - m_{K_2}$, etc., will be much too large, just as in the Georgi-Glashow 5-quark model. The Glashow-Iliopoulos-Maiani cancellation scheme can again be used to ameliorate this difficulty, most simply through the introduction of another doublet $(p', \lambda_c)_L$, as proposed by Weinberg.[12] Lee[17] and Prentki and Zumino[18] (LPZ) have shown that it is possible to eliminate both anomalies and lowest-order couplings to neutral neutrino currents, at the price of two additional quarks. The effective $(\bar{\lambda}n)(\bar{\mu}\mu)$ and $(\bar{\lambda}n)(\bar{\lambda}n)$ interactions that one computes in these models are essentially the same as in the Georgi-Glashow 8-quark model, and the constraints on quark masses are thus about the same. (The constraints in the LPZ model are almost exactly the same as in the 8-quark model; the constraints are weaker in Weingerg's model[12] by a small numerical factor because the smaller number of quarks decreases the number of contributing Feynman diagrams.)

In the LPZ model, there are two different Higgs' scalar fields responsible for generating the muon and neutral quark (n,λ) masses. It follows that the contribution of single Higgs' scalar exchange to processes like $K_L^o \to \bar{\mu}\mu$ or $K^+ \to \pi^+ e^+ e^-$ vanishes, insofar as we can neglect the mixing between these scalar fields generated by their interaction, and consequently no constraint on the masses of these neutral scalars is obtained. (The same conclusion was obtained, although for a different reason, in the Georgi-Glashow 8-quark model.) The masses of the two charged Higgs' scalars which are present in the LPZ model also appear to be essentially unconstrained by experiment.

We thus conclude that the constraints on the SO(3) models which we have obtained by considering $K_L \to \bar{\mu}\mu$, $m_{K_1} - m_{K_2}$, etc., are likely to apply fairly generally to models of weak interactions built on the principle of spontaneously broken gauge symmetry.

APPENDIX

In the Georgi-Glashow SO(3) models,[5] it is necessary either that the mass of the Higgs' scalar be large or that the $\bar{\lambda} n \phi$ coupling be small, in order that the predicted rate for $K^+ \to \pi^+ \bar{e} e$ not be too large. This coupling can, in fact, be shown to vanish in an 3-quark version of the O(3) model,[5] which also incorporates the Glashow-Iliopoulos-Maiani mechanism.[7]

In this model, the quarks are grouped into two SO(3) triplets ψ_1, ψ_2 and two singlets S_1, S_2, which we have chosen as follows:

$$\psi_1 = \begin{bmatrix} p \\ (n_c \sin\beta + q^0 \cos\beta)_L + q^0_R \\ q^- \end{bmatrix}, \quad \psi_2 = \begin{bmatrix} p' \\ (\lambda_c \sin\beta + q^{0\prime} \cos\beta)_L + q^{0\prime}_R \\ q^{-\prime} \end{bmatrix}, \tag{A1}$$

$$S_1 = (n_c \cos\beta - q^0 \sin\beta)_L + n_R ,$$
$$S_2 = (\lambda_c \cos\beta - q^{0\prime} \sin\beta)_L + \lambda_R ,$$

where p, n and λ are the uncharmed integrally-charged quarks, and

$$n_c = n \cos\theta_c + \lambda \sin\theta_c \,, \quad \lambda_c = -n \sin\theta_c + \lambda \cos\theta_c . \tag{A2}$$

Let us now show that the $\bar{\lambda} n \phi$ coupling vanishes in this model. The Lagrangian has the form

$$\mathcal{L} = \bar{\psi}\left(i\gamma\cdot\partial - \gamma\cdot \underset{\sim}{W}\cdot \underset{\sim}{T}\right)\psi - \psi^\dagger \beta M_0 \psi - \psi^\dagger \beta \underset{\sim}{\Gamma}\, \psi \cdot \underset{\sim}{\phi} \tag{A3}$$
$$+ \mathcal{L}_W + \mathcal{L}_\phi + \cdots ,$$

in a basis in which the quark mass matrix is diagonal; ψ represents all 8 quarks, W^o is the photon, $\phi^o = \phi + M_W/e$ is the unshifted Higgs' field, and β is the Dirac matrix γ_o. The matrix $\underset{\sim}{T}$ corresponds to the reducible representation of O(3) $3 \oplus 3 \oplus 1 \oplus 1$; including the charge in $\underset{\sim}{T}$, the algebra is

$$[T_+, T_-] = eT_0 \,, \quad [T_\pm, T_0] = \pm e T_\pm \,. \tag{A4}$$

The coupling matrix $\beta\underset{\sim}{\Gamma}$ transforms as a vector under O(3) and the bare mass term M_o is a singlet. Thus the full mass matrix is

$$M = M_0 + \underset{\sim}{\Gamma} \cdot \langle \underset{\sim}{\phi} \rangle = M_0 + \Gamma_0 M_W/e \,. \tag{A5}$$

It follows that

$$\beta\Gamma_0 = e^{-1}[T_+, \beta\Gamma_-] = -e^{-2}[T_+, [T_-, \beta M]]. \tag{A6}$$

The Glashow-Iliopoulos-Maiani mechanism cancels p′ against p and $q^{-\prime}$ against q^- intermediate states in order that

$$\langle \lambda | T_- T_+ | n \rangle = \langle \lambda | T_+ T_- | n \rangle = 0. \tag{A7}$$

Furthermore, since n_R and λ_R appear only in O(3) singlets in Eq. (A1), application of $T_\pm$ to $|n\rangle$ or $|\lambda\rangle$ can only result in left-handed objects. From this observation and Eqs. (A6) and (A7), it follows that

$$\begin{aligned} e^2 \langle \lambda | \Gamma_0 | n \rangle &= \langle \lambda | \beta (T_+ \beta M T_- + T_- \beta M T_+) | n \rangle \\ &= 0 \,, \end{aligned} \tag{A8}$$

since $(1-\gamma_5)\,\beta M(1-\gamma_5) = (1-\gamma_5)\,(1+\gamma_5)\beta M = 0$. (Note that if we had mixed n_R or λ_R with $q^{o\prime}_R$ in the second triplet, this final step would be invalid.)

FOOTNOTES

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[16] For example, T. Appelquist, J. D. Bjorken, and M. Chanowitz in a forthcoming paper note that the lowest SU(3) representation in which $\mathscr{L}_{\Delta S} = 2$ can lie is a 27, and they estimate the matrix element by comparison with $K^+ \to \pi^+ \pi^0$, which they assume is pure 27. The result is in rough agreement with ours. We would like to thank Professor Bjoken for telling us of this work.

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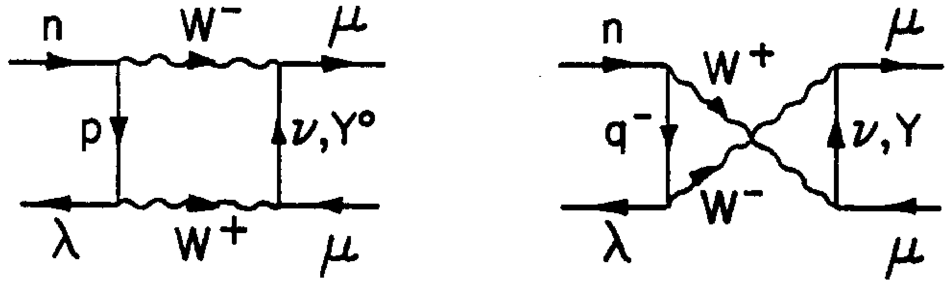

Figure 1

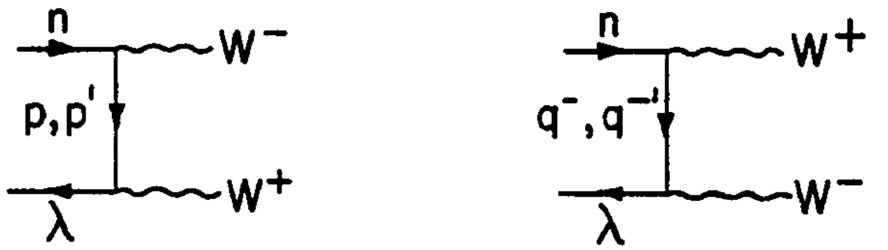

Figure 2

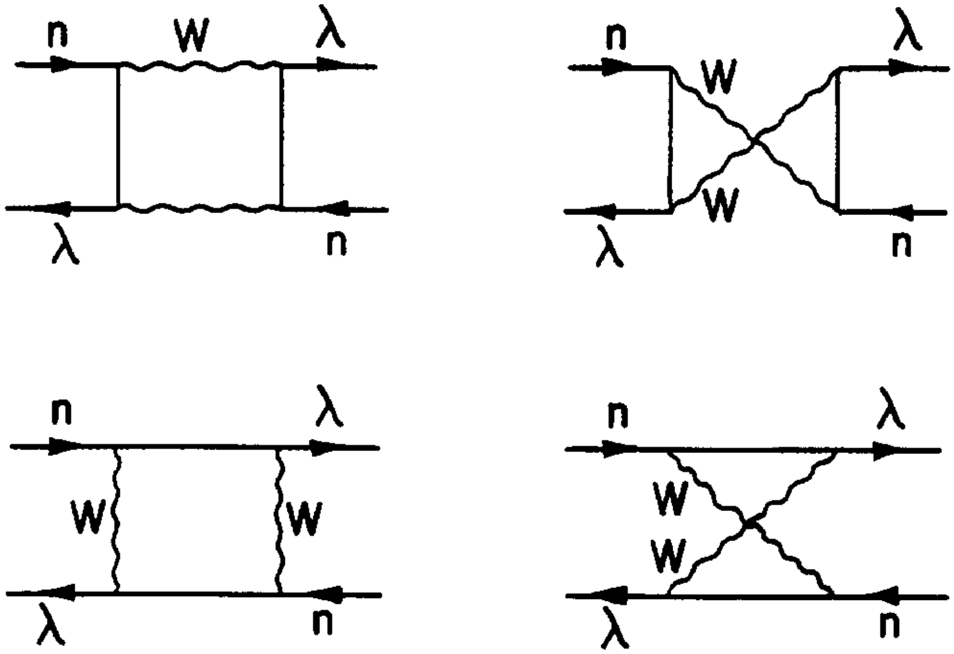

Figure 3